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An ideal mating surface method used for tolerance analysis of mechanical system under loading

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Abstract

A method to substitute the actual mating surfaces into an ideal mating surfaces is proposed in this paper. A unit normal vector is used to express their position and orientation. To simulate the variation propagation in assemble process, an error accumulate model was built in the foundational coordinate system and can be solved by the homogeneous transformation matrix (HTM). Thus the accuracy prediction of mechanical system could be realized in the condition of rigid body. The ideal matting surfaces under loading could be calculated by finite element method. The parameters of the normal vectors would be varied due to the part deformation. By discretization of vector elements in tolerance zone, the actual element variation under loading can be calculated and the distribution and probability density function compared to the rigid body can be obtained. A grind dress was taken as an example to illustrate this method.

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Keywords: tolerance analysis, small displacement torsor, assembly process, part deformation

1. Introduction

In machine tools and other high precision mechanical systems, the precision of parts significantly impacts on product performance. An effective model is needed to analyze system accuracy and determine the parts' accuracy considering a variety of requirements. However the traditional accuracy predicting methods have a tedious and error-prone calculation. More importantly, it separates the combined effect and interaction of dimensional tolerances and geometric tolerance in precision forming process. In recent years, many scholars had great achievement on the tolerance analysis of complex mechanical system. Alain[1] and Philippe[2] applied Jacobian matrix to establish the statistically tolerance analysis model. ZHOU[3] used several simulation generates pseudo-random number to improve the computational efficiency using Monte Carlo tolerance analysis complex assembly components. Zhang[4] established three levels statistical tolerance ring structure, proposed one statistical tolerance design method. Anselmetti[5] discussed a variety of surface tolerance chain. Zhang[6] used polychromatic sets to describe the feature-based hierarchical tolerance information, reasoned constraint meta-level of the underlying framework. Shen[7] comparative studied the currently four analytical methods.

This paper proposed the part model with dimensional tolerances and geometric tolerances information, from the changes range of ideal face. This paper extracted the unit normal vector of ideal surface, obtained the spatial location and distribution of the mechanical systems' ideal assembly plane by the homogeneous transformation matrix, then achieved the accuracy prediction of the whole mechanical system.

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2. Methodology

2.1. Model of the ideal surfaces

The ideal surfaces are the ideal planes of parts with assumed full contacting; they could produce the same effect of error propagation and accumulation as the actual surface. Ideal surface could use the geometric centre unit normal vector of the parts to represent the spatial position and orientation, includes angles (α,β,γ) and coordinates (x,y,z). For a single part, the Cartesian coordinate system could be established in one geometric centre of the ideal surface of parts. In this coordinate system, the unit normal vectors of the parts could be used to present the actual mating surfaces. Then the mathematical model of single part could be gotten for the accuracy prediction, including all geometry information.

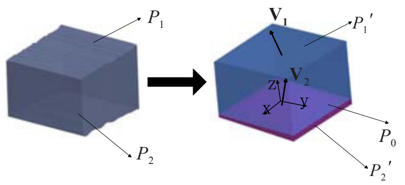

Fig. 1. The ideal surface of parts

In Figure 1, P_1', P_2' are the ideal surface corresponding, P_0 is the ideal location of P_2, the coordinate system origin located at the geometric center, $\mathbf{V_1}$, $\mathbf{V_2}$ are the unit normal vectors of P_1', P_2'. The unit normal vector of ideal surface representation is:

$$\mathbf{D} = [\mathbf{R};\mathbf{T}] = [\alpha,\beta,\gamma;x,y,z]^T$$

R is the rotation sub-matrix, including α, β and γ parameter, T is the displacement sub-matrices, including x, y and z parameters.

The variation range of unit normal vector parameters could express three-dimensional space change range constrained by tolerances. Firstly, the parameters of the unit normal vector could be gotten from surface design information conversion methods, and then the range of unit normal vector's variation could be obtained according to the tolerance principle of requirements.

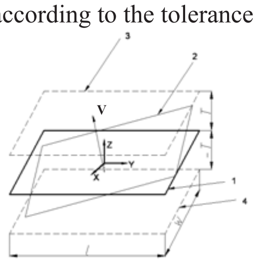

Fig. 2. The unit normal vector's range controlled by flatness tolerance

The plan 1 is the mating surface's ideal location, 2 is one ideal surface within the control of flatness, 3 and 4 are the variation range within the control of ideal flatness tolerance. $\mathbf{V}$ is the unit normal vector of 2, its sub-matrix form and the parameter variation are as follows:

$$\mathbf{R} = \begin{bmatrix}\Delta\alpha & \Delta\beta & 0\end{bmatrix}^T \quad (1)$$

$$\mathbf{T} = \begin{bmatrix}0 & 0 & \Delta z\end{bmatrix}^T \quad (2)$$

$$-\frac{2T}{l} \le \Delta\alpha \le \frac{2T}{l} \quad (3)$$

$$-\frac{2T}{w} \le \Delta\beta \le \frac{2T}{w} \quad (4)$$

$$-T \le \Delta z \le T \quad (5)$$

$$-T \le \Delta z + \frac{l}{2}\times\Delta\alpha + \frac{w}{2}\times\Delta\beta \le T \quad (6)$$

$\Delta\alpha$ and $\Delta\beta$ are the deflection angles that $\mathbf{V}$ around the axis x and the axis y, Δz is the change value V's starting point along the axis. $2T$ is the value of flatness tolerance, l is the length, w is the width.

2.2. Variation propagation of assembly processes

The geometry error of feature during assembly will be converted to the unit normal vectors in the part coordinate systems, by using the homogeneous transformation matrix to embed the unit normal vectors in the assembly path. The error accumulation model in the assembly process could be gotten.

In a single coordinate system, define the unit normal vector from the origin to point of position vector, convert it to a reference coordinate system:

$$\mathbf{T}' = \mathbf{R_M}\mathbf{T} + \mathbf{T_M} \quad (7)$$

$\mathbf{T}$ is the position vector in part coordinate system, $\mathbf{T_M}$ is the displacement matrix between the coordinate systems, $\mathbf{R_M}$ is the rotation matrix, M is the transformation matrix.

$$\begin{bmatrix}\mathbf{T}' & 1\end{bmatrix}^T = \mathbf{M}\begin{bmatrix}\mathbf{T} & 1\end{bmatrix}^T \quad (8)$$

$$\mathbf{M} = \begin{bmatrix}\mathbf{R_M} & \mathbf{T_M} \\ 1 & 1\end{bmatrix} \quad (9)$$

$$\mathbf{R_M} = \begin{bmatrix}1 & -\gamma_M & \beta_M \\ \gamma_M & 1 & -\alpha_M \\ -\beta_M & \alpha_M & 1\end{bmatrix} \quad (10)$$

$$\mathbf{T_M} = \left[x_M, y_M, z_M\right]^T \quad (11)$$

α_M , β_M and γ_M are the deflection angle of the part, relative to reference coordinate system. x_M、y_M and z_M are the values of the offsets along the X, Y and Z axis, relative to the reference coordinate system .

$$\mathbf{C}' = \mathbf{R_M}\mathbf{C} \tag{12}$$

$$\mathbf{C} = \left[\cos\alpha, \cos\beta, \cos\gamma\right]^T \tag{13}$$

C is the direction cosine matrix, α 、 β and γ are the angle of the unit normal vector in parts coordinate system.

For the ideal mating surface method, the unit normal vectors of part's ideal mating surface should be created. They could express the connection of unit normal vectors in different part coordinate systems.

Shown in Figure 3, P_I and P_J are the ideal face, their unit normal vectors are $\mathbf{D_{IO}}$ and $\mathbf{D_{IO}}$ in O_I . $\mathbf{D_{FO}}$ and $\mathbf{D_{FJ}}$ are the unit normal vectors of P_{FJ} in O_I and O_J . $\mathbf{M_{IJ}}$ is the transformation matrix.

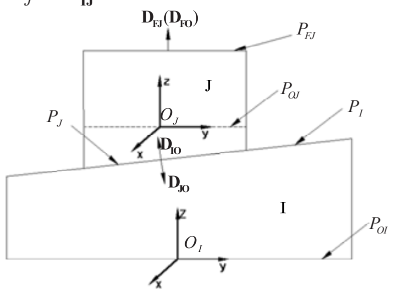

Fig. 3. Unit normal vectors transformation

$$\left[\mathbf{T_{FO}} \quad 1\right]^T = \mathbf{M_{IJ}}\left[\mathbf{T_{FJ}} \quad 1\right]^T \tag{14}$$

$$\mathbf{C_{FO}} = \mathbf{R_M}\mathbf{C_{FJ}} \tag{15}$$

$$\mathbf{M_{IJ}} = \left[\mathbf{D_{IO}} \quad \mathbf{I}\right]^T - \left[\mathbf{D_{JO}} \quad \mathbf{I}\right]^T \tag{16}$$

In Figure 4, the mechanical system is assembled by m parts, $\mathbf{M_{IK}}$ is the transformation matrix between part $K-1$ and K . $\mathbf{D_m}$ is the unit normal vector of functional surface of the end part M . $\mathbf{D_m}'$ is the unit normal vector in the base coordinate system, can be calculated by the following formula:

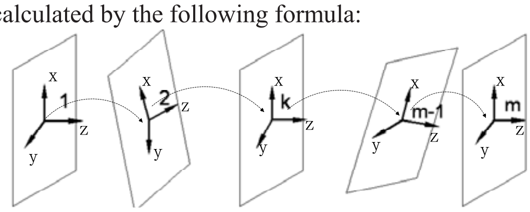

Fig. 4. Mechanical system assembly structure

$$\left[\mathbf{T_m}' \quad 1\right]^T = \prod_{k=2}^{m}\mathbf{M_{IOK}}\left[\mathbf{T_m} \quad 1\right]^T \tag{17}$$

$$\mathbf{C_m} = \prod_{k=2}^{m}\mathbf{R_M}\mathbf{C_m} \tag{18}$$

2.3. Variation of ideal surface under loading

When the working condition is considered, the position and orientation changes of the vectors of the ideal surface due to part deformation should be calculated.

Firstly, the discrete elements of unit normal vector in variation zone could be picked out to simulate the geometry trend. The geometry model could be built according to these discrete elements. The actual feature changing under loading can be obtained account into FEM-based approaches. The mapping function between corresponding elements before and after loading is established by the independent variables of elements after loading. Then the probability density function after loading can be obtained by substitution of the mapping function into former probability density function, and this probability density function also expresses the error distribution after loading. This process is shown in Fig. 5.

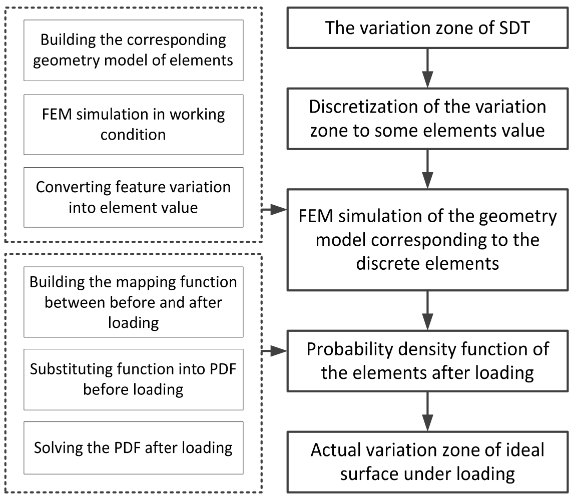

Fig. 5. Process of simulation of variation of ideal surface under loading

3. Result and Discussion

3.1. Tolerance analysis based on the ideal surface model

Taking grinding dresser's feeding system as an example, the process of accuracy prediction will be illustrated by the ideal surface method. Shown in Figure 6, the dresser is assembled by six parts. The location of part 6 reflects the final assembles precision in the base coordinate system.

Table 1. The tolerance of surfaces

Mating surface	Flatness /mm	Mating surface	verticality /mm	Mating surface	Parallelism/mm
A	0.02	A	0.005	B1	0.005
B	0.02	N	0.005	L1	0.005
N	0.02	L	0.005	N1	0.005
L	0.02	F	0.001	I	0.001
E	0.005	G	0.001	F	0.001
I	0.005				

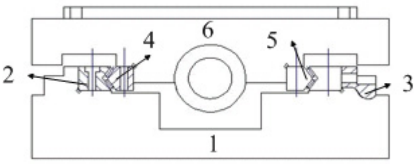

Fig. 6. Dresser feeding system

The structure and the relationship of various parts are shown in Figure 7 and 8. The surfaces' shape and position tolerances are described as Table 1.

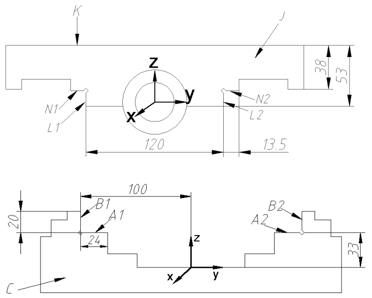

Fig. 7. Base and Worktable

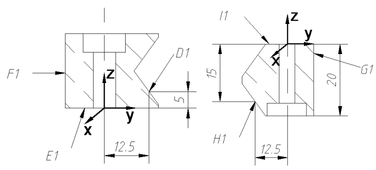

Fig. 8. Slideway

The ideal unit normal vector's location coordinates is (0, -92.5,33), with the constraints as (19) to (25) to control the variation range of parameters.

$$\frac{-0.01}{24} \le \Delta\alpha \le \frac{0.01}{24} \quad (19)$$

$$\frac{-0.01}{300} \le \Delta\beta \le \frac{0.01}{300} \quad (20)$$

$$-0.01 \le \Delta z \le 0.01 \quad (21)$$

$$-0.01 \le \Delta z + \Delta\alpha \times 12 + \Delta\beta \times 150 \le 0.01 \quad (22)$$

$$\frac{-0.025}{24} \le \Delta\alpha \le \frac{0.025}{24} \quad (23)$$

$$-0.025 \le \Delta z \le 0.025 \quad (24)$$

$$-0.025 \le \Delta z + \Delta\alpha \times 12 \le 0.025 \quad (25)$$

The actual assembly process is: firstly part 2 and 3 join part 1, secondly parts 4 and part 5 join 6. Finally the whole assembly could be gotten. The coordinate system of part 1 is the mechanical system's reference, the ideal surfaces of part 2 and part 3 will be converted to the base coordinate system, then the ideal surface 2 * is overall fitted from parts 1,2 and 3. The ideal surface 4 * could be gotten by the same method. Finally it is converted to the basic coordinate system of part 1.

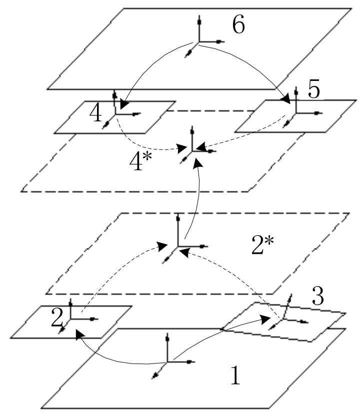

Fig. 9. Precision analysis structure of dresser

3000 random error samples were selected from the variation range of unit normal vector in normal distribution to simulate numerical analysis the assembly precision. The contours of P position in the reference coordinate system could be gotten.

The space coordinates of point P could be gotten after 3,000 times accuracy analysis simulation. The average value of position error is 57.1, and the variance is 46.8.

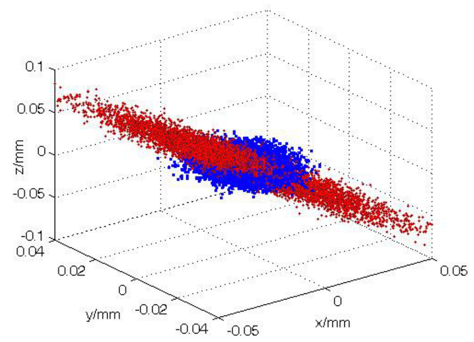

Fig. 10. The location of working point

3.2. Under working condition

According to the method mentioned in 2.3, based on the simulation on nominal size of FEA model under loading, the corresponding tolerance zone and distribution variation could be analysed in the following processes.

As showed in Fig. 11, taking the α element of unit normal vector in the A surface of base part as an example.

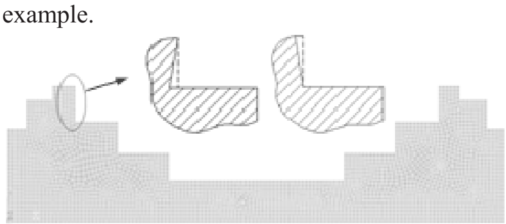

Fig. 11. The variation of angle element α of A surface

A rigid surface was modeled and moved close to the contact surface to generate a displacement constraint. The mapping curve between displacement and reaction force can be obtained by FEA approach. Conversely, according to the actual loading on the fitting surface corresponded to the displacement of the rigid surface, the variation of the corresponding element can also be calculated, and the multi-loading condition can be simulated in the same way.

As shown in Fig. 12, the variation zone of angle element α of A surface is dispersed into some points. The relationship between the displacement of the rigid surface and the reaction force of the fitting surface is illustrated.

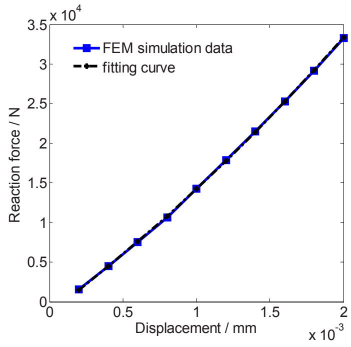

Fig. 12. The relationship between displacement and the reaction force

Based on the displacement-reaction force curve, the actual fitting surface deformation under loading can be simulated. By fitting the nodes coordinate after deformation using ideal surface, the α element can be obtained.

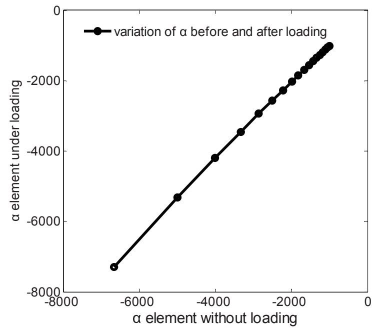

Fig. 13. Variation of α element before and after loading

From the ideal surface fitting the deformation part, the corresponding function between α element without or under loading can be got. By substituting this function into probability density function without loading, the probability density distribution under loading can be solved.

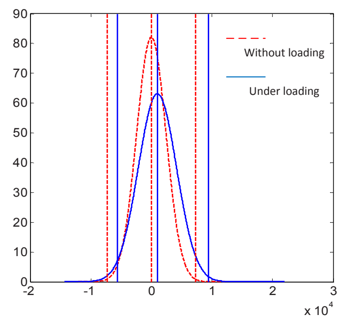

Fig. 14. The probability density distribution before and after loading

Based on the variation zone and distribution of unit normal vector of the ideal surface calculated in this method, the tolerance analysis of the assembly under loading would be more easily.

The grinding dresser is taken as the example. Considering the normal work condition, the error samples are obtained according to the tolerance specification, and revised through FEM approach. The assembly accuracy is predicted by the variation propagation model. In this example, 3000 samples were picked to simulate the normal distribution of the geometry tolerance of each feature.

Comparing the two figures in Fig. 15, the distribution and probability density function of the working point under loading is different to the former simulation without considering part deformation.

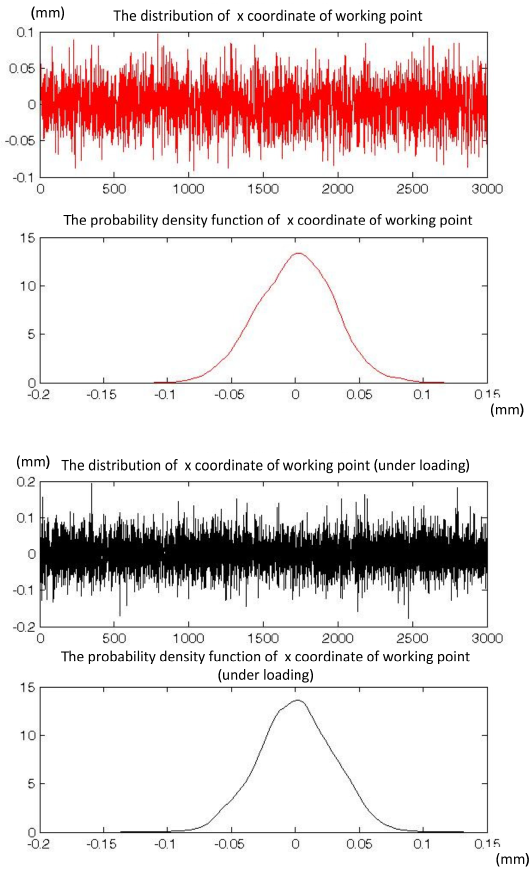

Fig. 15. The distribution and probability function of working point without and under loading

4. Conclusions

(1) This paper proposed a part precisions model covered dimensional tolerances and geometric tolerances information.

(2) The method of unit normal vector's variation is proposed in the rigid condition. The variation range and distribution of unit normal vector by the load is discussed.

(3) The accumulation error model is established based on ideal mating surface method. It realizes the accuracy prediction of the mechanical system. The accuracy prediction could be calculated by selecting the samples. The samples could be gotten by the variation range and distribution of unit normal vector.

(4) The distribution would be different as the part deformation under loading was considered. As the error variation is much less than the nominal dimension, it is not a significant difference.

Acknowledgements

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